TWO PHASE FLOW PRESSURE DROP AND HEAT TRANSFER IN A MULTI-SLOT INSERT IN THE ENTRANCE OF A BOILER TUBE

Aerojet General Contract No. OP 119224

GLM-61

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November 1967

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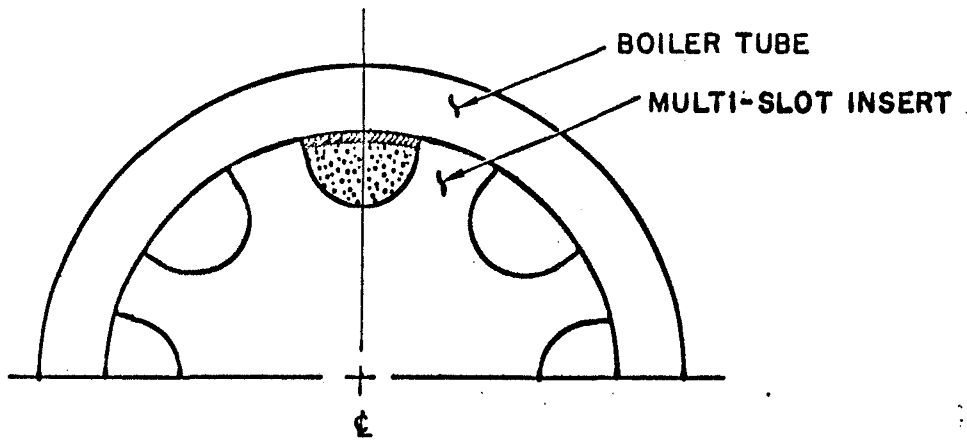
I. INTRODUCTION

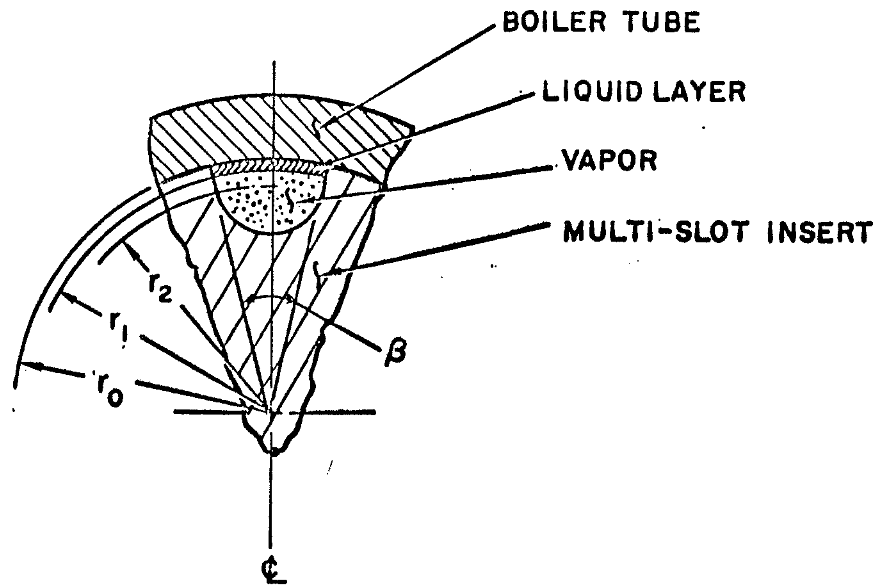
It is desired to estimate both heat transfer and pressure drop characteristics of the multi-slot insert at the entrance of the Aerojet mercury boiler. Saturated liquid enters the slotted insert and changes to a two-phase mixture as it passes through this region. Because the slots are machined in a helical fashion on the periphery of the insert rod, it is believed that the two phases will readily be separated by the centrifugal force field. It is postulated that a thin, liquid layer exists at the outer slot region contiguous to the boiler tube wall and that the remainder of the slot is filled with vapor (see Figure 1).

The pressure drop and heat transfer analyses to be presented in the next section for this system closely follow procedures previously used at Geoscience to study separated, annular flow in circular cross sectional tubes (Reference 1). It seems worthwhile to review how well these previous annular models predicted actual pressure drop and heat transfer behavior.

The frictional pressure drop model for a viscous-liquid annulus at the wall of a circular cross sectional tube with a turbulent vapor core was used to predict pressure drop and void fraction in experimental flow systems where annular patterns have actually been observed. Figure 2 shows a comparison of the void fractions calculated for the viscous-turbulent model with several sets of experimental data (References 2 and 3). Figure 3 presents comparisons for pressure drop. The predictions were based on an interface friction factor of 0.10. Note the reasonable agreement between predicted and experimental void fraction values over a nearly thousandfold range of the parameter $(G_\ell \mu_\ell \rho_v)/(G_v \mu_v \rho_\ell)$. The experimental pressure drop data scatter significantly but in general fall above the predictions.

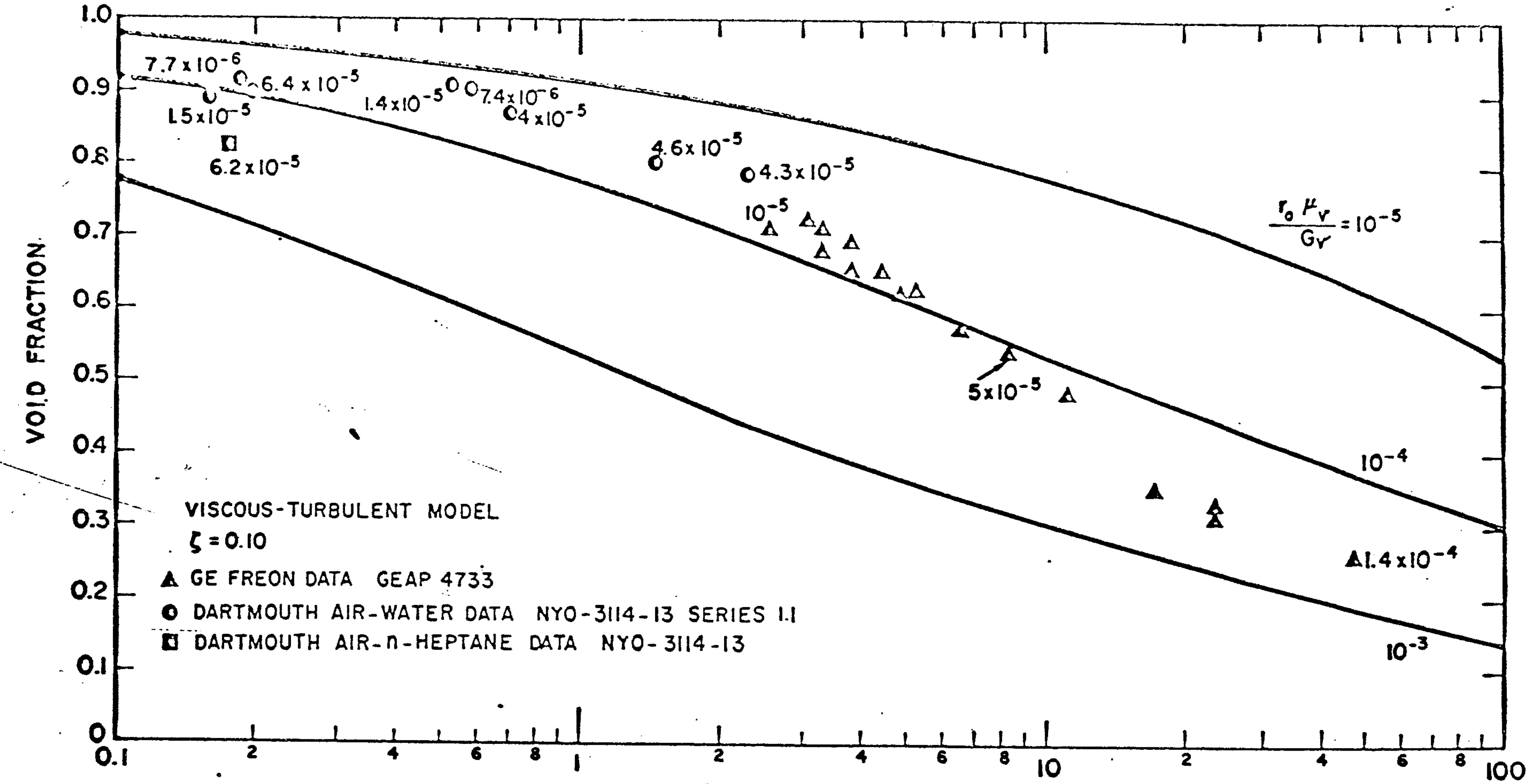
The annular heat transfer model was also used to predict low vapor quality boiling heat transfer when the liquid phase was in direct contact with the wall in the form of an annulus. Some of the linear flow boiling potassium heat transfer data obtained by the General Electric Company (Ref. 4) in their 100 kw system are shown plotted in





SECTION OF ONE SLOT

Figure 1. Schematic diagram of an idealized multi-slot insert.



$$\frac{G_\ell \mu_\ell \rho_v}{G_v \mu_v \rho_\ell}$$

$$\frac{r_o \mu_v}{G_v} = 10^{-5}$$

Figure 2. Comparison of predicted and measured void fractions in annular flow.

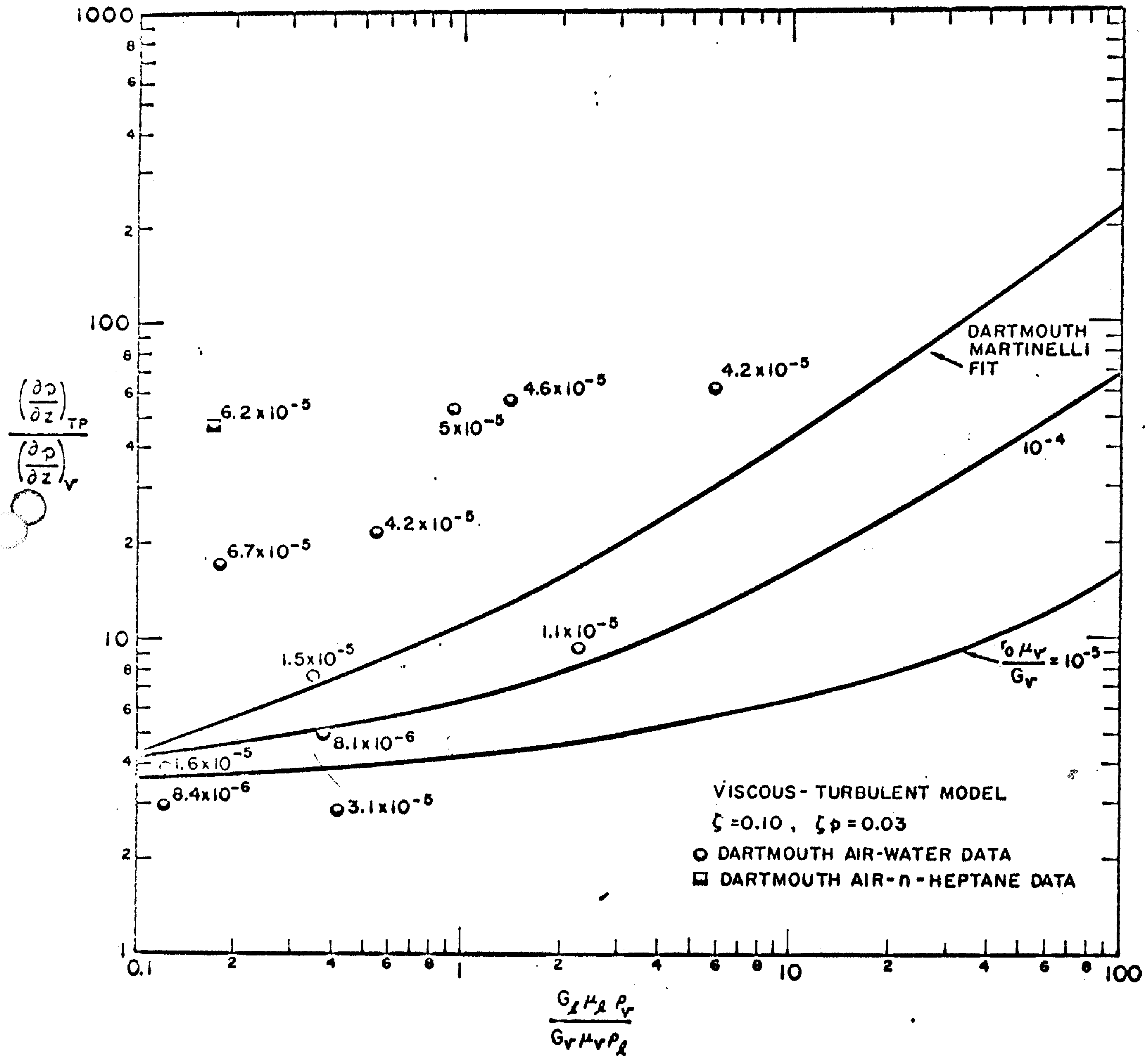


Figure 3. Comparison of predicted and measured two phase flow pressure drops in annular flow.

Figure 4. The experimental conductances are presented as a function of vapor quality, flow rate and pressure level. These data fall above a number of the other sets of boiling liquid metal data. As a matter of fact, these conductances appear to increase with vapor quality (in the low vapor quality region) much in the same way as the conductances do for forced flow boiling water. The three superheated liquid film models to be described later are also compared to the experimental General Electric data in Figure 4. In all cases the liquid-vapor interface temperature was postulated to be a fixed saturation temperature . The predicted curves for the heat transfer conductances as a function of vapor quality for the three models are noted to have the same shape as the experimental General Electric data. The models describing volumetric boiling (particularly, the linear heat sink distribution) fall closest to the experimental measurements.

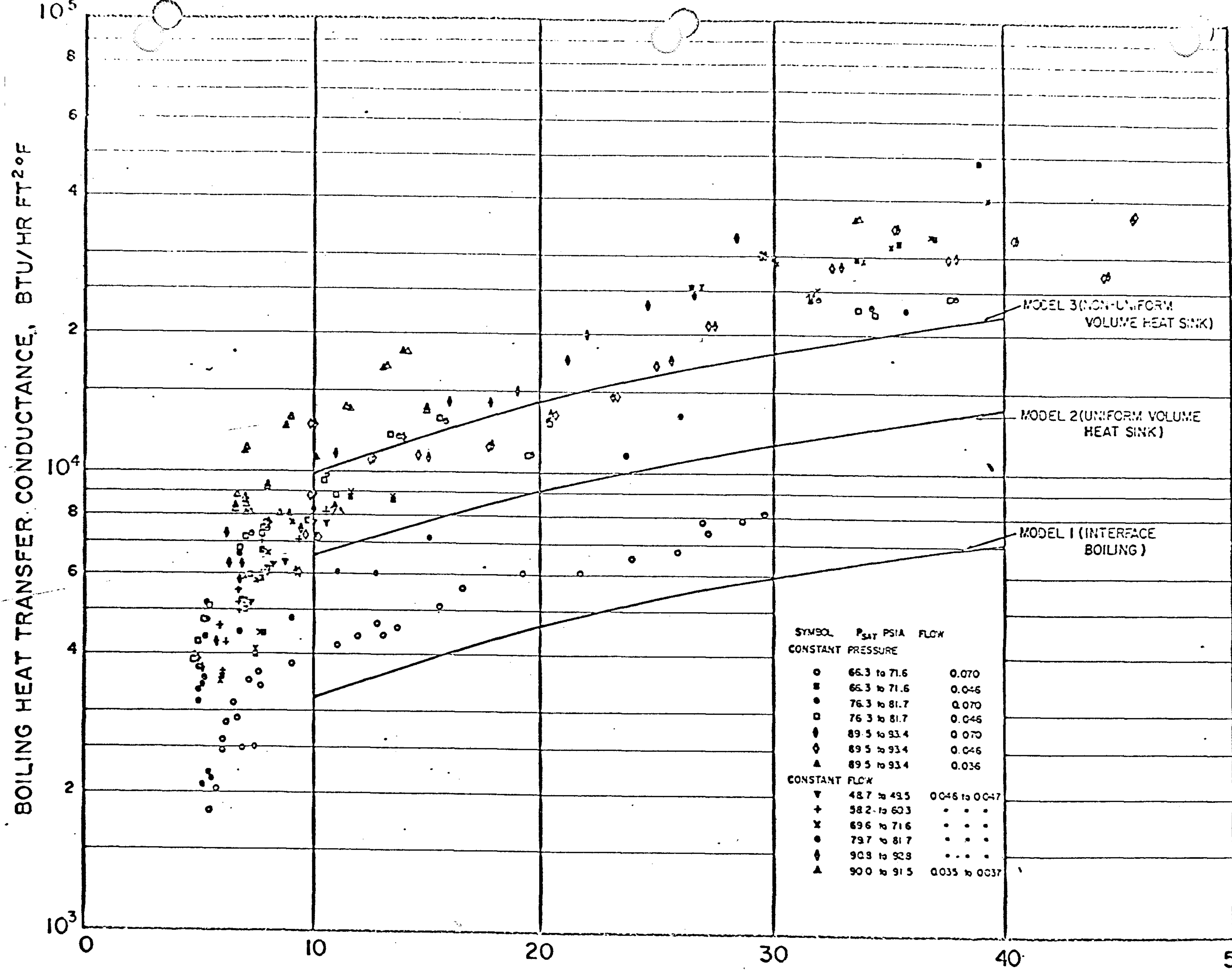


Figure 4. Comparison of superheated liquid film boiling models with GE boiling potassium data.

II. ANALYSES

A. Frictional Pressure Drop

From the shear stress-pressure drop and shear stress-strain equations one can obtain an equation for the velocity profile in the liquid layer, that is, for $r_o < r < r_1$,

$$v = -\frac{1}{2\mu_o}\frac{\partial p}{\partial x}\left(\frac{r^2}{2} - \frac{r_o^2}{2}\right) \tag{1}$$

Upon integrating from r_o to r_1, the liquid mass flow rate, G_o, can be obtained,

$$G_o = 2\pi w_o\left[-\frac{1}{4\mu_o}\frac{\partial p}{\partial x}\left(\frac{r_o^4}{4} - \frac{r_1^4}{4}\right) + \frac{r_o^2}{4\mu_o}\frac{\partial p}{\partial x}\left(\frac{r_o^2}{2} - \frac{r_1^2}{2}\right)\right]\frac{\beta}{2\pi} \tag{2}$$

This relation represents one equation in two unknowns, namely, r_1 and $(\partial p)/(\partial x)$,

where v, velocity profile in liquid layer

μ_o, liquid viscosity

$\frac{\partial p}{\partial x}$, two phase axial frictional pressure gradient in liquid and vapor

r, radial distance

r_o, inside radius of tube

r_1, radial distance to interface between two phases

G_o, liquid mass flow rate

w_o, liquid weight density

β, slot angle shown in Figure 1, radians

The pressure drop equation for turbulent gas flow in the irregular cross section is,

$$\frac{\partial p}{\partial x} = \frac{\zeta\, w_1}{4R_h} \frac{V^2}{2g} \tag{3}$$

where ζ, Weisbach friction factor for interface between the two fluids

w_1, vapor weight density

V, mean vapor velocity

g, acceleration of gravity

R_h, hydraulic radius

In this analysis, it is postulated that the viscous liquid-turbulent vapor interface velocity, v_2, is small compared to the mean vapor velocity, V, and is thus neglected. The hydraulic radius definition is*

$$R_h = \frac{\text{flow area}}{\text{wetted parameter}} = \frac{\pi\left(r_1^2 - r_2^2\right)\frac{\beta}{2\pi} + \frac{\pi}{8}\left(\beta\, r_2\right)^2}{\beta\, r_1 + 2\left(r_1 - r_2\right) + \frac{\pi}{2}\beta\, r_2} \tag{4}$$

where r_2 is the radial distance to the center of the arc defining the inner bound of the slot. The mean vapor velocity squared, V^2, can be expressed in terms of the continuity equation,

$$V^2 = \frac{G_1^{\;2}}{(\text{flow area})^2\, w_1^{\;2}} = \frac{G_1^{\;2}}{w_1^{\;2}\left[\left(r_1^{\;2} - r_2^{\;2}\right)\frac{\beta}{2} + \frac{\pi}{8}\left(\beta\, r_2\right)^2\right]^2} \tag{5}$$

Substitution of Equations (4) and (5) into (3) yields

$$\frac{\partial p}{\partial x} = \frac{\zeta\, w_1}{8g}\, \frac{G_1^{\;2}\left[\beta\, r_1 + 2\left(r_1 - r_2\right) + \frac{\pi}{2}\beta\, r_2\right]}{w_1^{\;2}\left[\left(r_1^{\;2} - r_2^{\;2}\right)\frac{\beta}{2} + \frac{\pi}{8}\left(\beta\, r_2\right)^2\right]^3} \tag{6}$$

*The geometrical bounds of the idealized slot are arcs of circles and radial segments. These arbitrary choices were made to simplify the algebra.

This expression is the second equation in two unknows, r_1 and $(\partial p)/(\partial x)$. Upon equating the pressure drop terms in Equations (2) and (6), there results,

$$\frac{G_o}{2\pi w_o\left[-\frac{1}{4\mu_o}\left(\frac{r_o^4}{4}-\frac{r_1^4}{4}\right)+\frac{r_o^2}{4\mu_o}\left(\frac{r_o^2}{2}-\frac{r_1^2}{2}\right)\right]\frac{\beta}{2\pi}} =$$

$$\frac{\zeta w_1}{8g}\,\frac{G_1^2\left[\beta r_1+2\left(r_1-r_2\right)+\frac{\pi}{2}\beta r_2\right]}{w_1^2\left[\left(r_1^2-r_2^2\right)\frac{\beta}{2}+\frac{\pi}{8}\left(\beta r_2\right)^2\right]^3} \tag{7}$$

or

$$\left[\frac{64}{\zeta\beta}\right]\left[\frac{g\,r_o\mu_1}{G_1}\right]\left[\frac{G_o\,\mu_o\,w_1}{G_1\,\mu_1\,w_o}\right]\frac{\left[(\rho^2-\rho_2^2)\frac{\beta}{2}+\frac{\pi}{8}(\beta\rho_2)^2\right]^3}{\left[\beta\rho+2(\rho-\rho_2)+\frac{\pi}{2}\beta\rho_2\right]} = \left[-\frac{1}{2}(1-\rho^4)+(1-\rho^2)\right] \tag{8}$$

where $\rho = r_1/r_o$ and $\rho_2 = r_2/r_o$. From Equation (7) it is possible to calculate r_1 (or ρ) the unknown liquid-vapor interface radius. This solution can be expressed symbolically as

$$\rho = \phi\left(\zeta, \beta, \rho_2, \frac{g\,r_o\,\mu_1}{G_1}, \frac{G_o\,\mu_o\,w_1}{G_1\,\mu_1\,w_o}\right) \tag{9}$$

The two phase flow frictional pressure drop, $(\partial p)/(\partial x)$, can be generalized as follows. The frictional pressure drop in the slot if completely filled with the vapor at flow rate, G_1, (no liquid present) would be,

$$\left(\frac{\partial p}{\partial x}\right)_{\text{all vapor}} = \frac{\zeta_s w_1}{8g}\;\frac{G_1^2\left[\beta r_o+2(r_o-r_2)+\frac{\pi}{2}\beta r_2\right]}{w_1^2\left[\left(r_o^2-r_2^2\right)\frac{\beta}{2}+\frac{\pi}{8}(\beta r_2)^2\right]^3} \tag{10}$$

Upon dividing Equation (6) by (10) one obtains

$$\frac{\frac{\partial p}{\partial x}}{\left(\frac{\partial p}{\partial x}\right)_{\text{all vapor}}} = \frac{\zeta}{\zeta_s}\left[\frac{\beta\rho + 2(\rho - \rho_2) + \frac{\pi}{2}\beta\rho_2}{\beta + 2(1-\rho_2) + \frac{\pi}{2}\beta\rho_2}\right]\left[\frac{(1-\rho_2^{\,2})\frac{\beta}{2} + \frac{\pi}{8}(\beta\rho_2)^2}{(\rho^2 - \rho_2^{\,2})\frac{\beta}{2} + \frac{\pi}{8}(\beta\rho_2)^2}\right]^3 \tag{11}$$

where ρ is already known from Equation (9).

B. Heat Transfer

In order to estimate boiling heat transfer in the low vapor quality region in the boiler tube insert with wetted walls, three analyses have been made for the heat transfer in idealized, liquid layers contiguous to a wall. The liquid layers are either viscous in character or are liquid metals. The first model consists of a superheated liquid layer that is heated from the tube wall and loses heat from the liquid-vapor interface by evaporative heat transfer; heat transmission through the layer is achieved by conduction only. The second model is defined by a superheated liquid layer with wall heat addition and a uniform volume heat sink representing volume boiling. The third model is defined by a superheated liquid layer with wall heat addition and a volume heat sink that varies linearly from a maximum value at the wall to zero at the liquid-vapor interface. No liquid-vapor interface evaporative heat transfer occurs in the last two models.

The general differential equation that defines these three boundary value problems is

$$u_m \frac{\partial t}{\partial z} = a \frac{\partial^2 t}{\partial y^2} - \frac{S}{\rho_\ell c_\ell} \tag{12}$$

where y is the transverse or radial dimension and z is the longitudinal dimenions. In all cases, $(\partial t)/(\partial z) = 0$. For the first system $S = 0$, for the second system $S = S_m$ and for the third system $S = S_o (1 - y/\delta)$ where $\delta = r_o - r_1$. A temperature solution was derived for each case. Next, the difference between the wall and liquid-vapor interface temperatures was substituted into the Nusselt modulus equation. The results are given in Table 1.

Momentum transfer equations for two phase flow in the insert have been derived in the previous section. From this theory it is possible to calculate liquid layer thicknesses vs vapor quality. Consequently, for a given vapor quality the Nusselt modulus and heat transfer coefficient are determined

Table 1.

Model	Nusselt Modulus
1. Heat loss at liquid-vapor interface	$\frac{2r_o}{\delta}$
2. Uniform volume heat sink	$\frac{4r_o}{\delta}$
3. Heat sink that varies linearly from maximum value at the wall to zero at the liquid-vapor interface	$\frac{6r_o}{\delta}$

III. EVALUATION OF PRESSURE DROP SOLUTION

Equations (9) and (11) have been evaluated over a portion of the parameter ranges involved, namely, $(g\, r_o\, \mu_1)/G_1$ and $(G_o\, \mu_o\, w_1)/(G_1\, \mu_1\, w_o)$ for $\zeta = 0.15$, $\zeta_s = 0.03$, $\beta = 0.245$, $\rho_2 = 0.9049$. The results are shown in Figures 5 and 6.

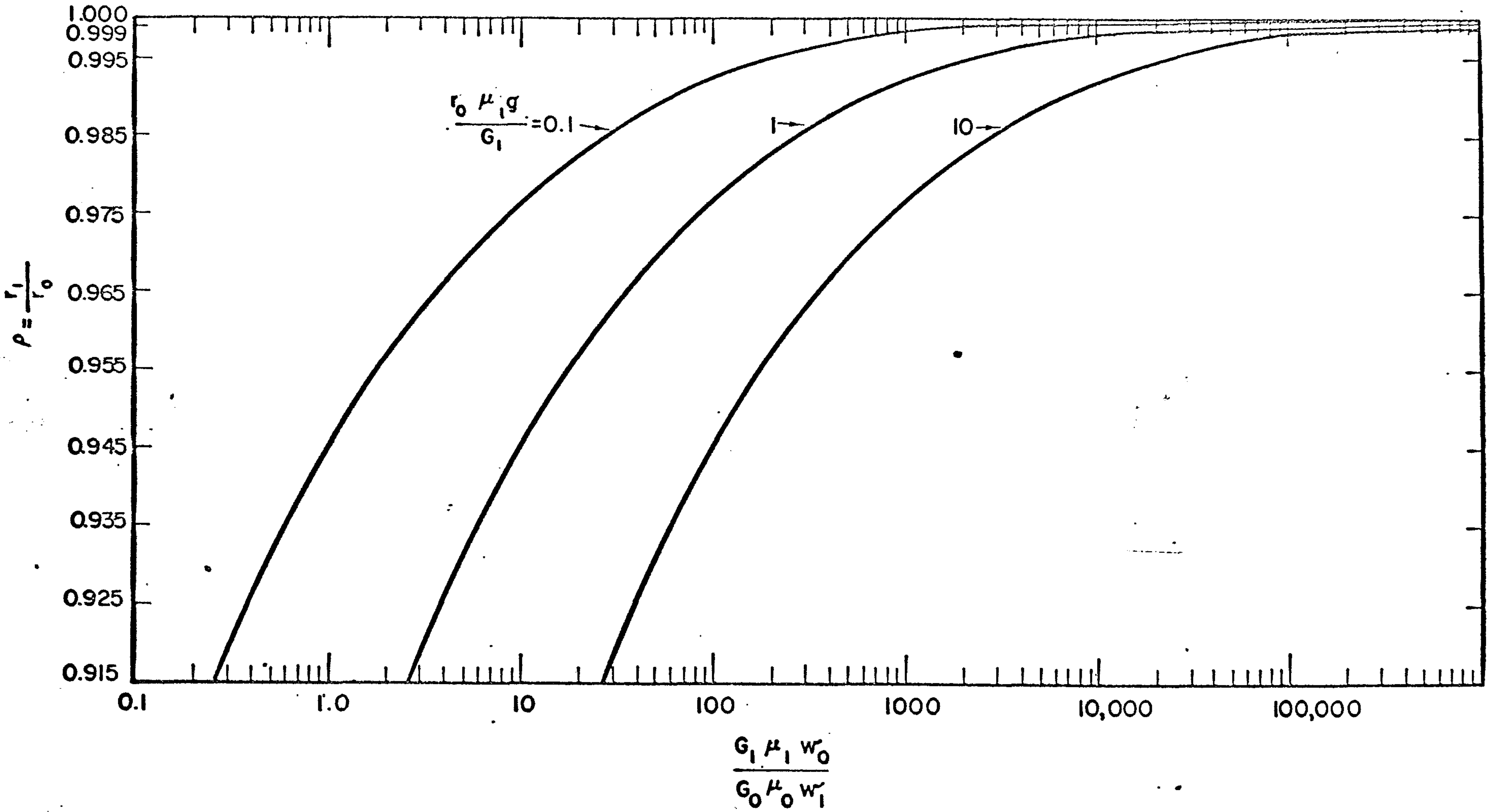


Figure 5. Radial liquid-vapor interface ρ for viscous-turbulent two phase flow in the slot with $\zeta=0.15$, $\zeta_s=0.03$, $\beta=0.245$ radians.

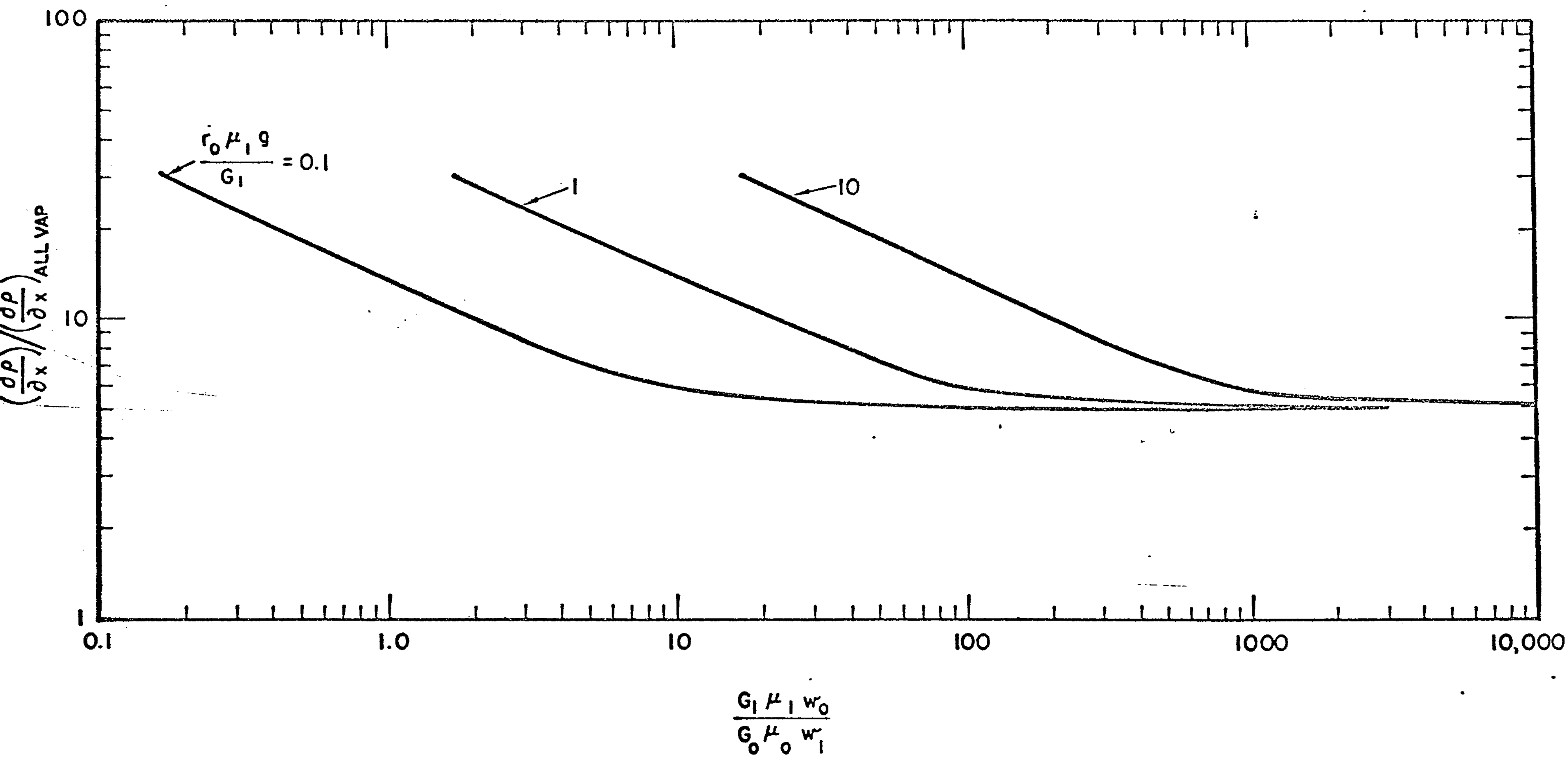


Figure 6. Viscous-turbulent two phase flow pressure drop in the slot with $\zeta = 0.15$, $\zeta_s = 0.03$, $\beta = 0.245$ radians.

IV. DISCUSSION

The frictional pressure drop solution presented in this memorandum can be used to calculate the overall pressure drop across the Aerojet entrance insert by the use of iterative integral methods. A simplified integral technique for a uniform wall heat flux case is presented in a previous Geoscience report (Reference 1) wherein the frictional pressure drop is related to vapor quality. Total pressure losses are obtained by summing the corresponding frictional and momentum change terms.

It is clear that when the viscous liquid layer contiguous to the boiler tube wall becomes very thin, it will break down into rivulet or droplet flow. The point at which this mechanism takes place is dependent, in a complicated way, on the physical properties, the flow state, and system geometry. This problem is currently under study at Geoscience.

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